

## Simple and Compound Interest

The lending and borrowing of money has been happening since thousands of years. Any sum of money, borrowed for a certain period, will invite an extra cost to be paid on the money borrowed; this extra cost at a fixed rate is called interest. The money borrowed is called principal. The sum of interest and principal is called the amount. The time for which money is borrowed is called period.

### Amount = Principal + Interest

The interest paid per hundred (or percent) for a year is called the rate percent per annum. The rate of interest is almost always taken as per annum, in calculations we will always consider it per annum unless indicated.

The interest is of two types, one is simple, the other is compound:

### Simple interest

It is the interest paid as it falls due, at the end of decided period (yearly, half yearly or quarterly), the principal is said to be lent or borrowed at simple interest.

Simple Interest,  $SI = PRT / 100$

Here P = principal, R = rate per annum, T = time in years

Therefore Amount,  $A = P + PRT/100 = P [1 + ( RT / 100 )]$

If T is given in months, since rate is per annum, the time has to be converted in years, so the period in months has to be divided by 12. if T = 2 months = 2/12 years)

**Example 1:** Find the amount on S.I. when Rs 4000 is lent at 5 % p.a. for 5 years.  
By the formula,  $A = P (1 + RT/100) = 4000( 1 + 5 \times 5/100 ) = Rs 5000$

### Compound Interest

The compound interest is essentially interest over interest. The interest due is added to the principal and that becomes the new principal for the interest to be levied. This method of interest calculation is called compound interest, this can be

for any period (yearly, half yearly or quarterly) and will be called “Period compounded” like Yearly compounded or quarterly compounded and so on.

First period’s principal + first period’s interest = second period’s principal

Compound interest = principal  $\{1 + \text{Rate}/100\}$ time - Principal

$$CI = P \{ 1 + R/100 \}^T - P$$

Here Amount = principal  $\{1 + \text{Rate}/100 \}$ time

**Example 2:** Find the compound interest on Rs 4500 for 3 years at 6 % per annum  
Using the formula,  $A = P (1 + R/100)^T = 4500(1 + 6/100)^3 = 4500 (1.06)^3 = 5360$   
Compound interest =  $5360 - 4500 = \text{Rs } 860$

## THE RULE OF 72

The rule of 72 is a quick way to show how long it will take to double your money under

The equation for the rule of 72 is:

Number of years for money to double =  $(72/\text{Annual Interest Rate})$  interest rate

At 8% interest, it will take  $72/8 = 9$  years for your money to double.

Here are more examples:

At 6%, it will take 12 years ( $72/6 = 12$ )

At 12%, it will take 6 years ( $72/12 = 6$ )

The rule of 72 is a short cut to estimate the magic of compound interest that makes your money grow.

- Remember that the rule of 72 is an approximation and its accuracy reduces as the interest rate becomes high.

## Important notes

1. In case interest is paid half yearly, then the interest is divided by 2, and used as  $(R/2)$  in the formula and the time is multiplied by 2, and used as  $2T$  in the formula, given by  $A = P [ 1 + ( R / 200 ) ]^{2T}$

**Example 3:** Find the compound interest on Rs 5000 for 3 years at 6 % per annum compounded half yearly.

$$\begin{aligned} \text{Using the formula, } A &= P [ 1 + ( R / 200 ) ]^{2T} \\ &= 5000(1 + 6/200)^{3 \times 2} \end{aligned}$$

$$= 5000 (1.03)^6 = 5971$$

$$\text{Compound interest} = 5971 - 5000 = \text{Rs } 971$$

**2.** In case interest is paid quarterly, then the interest is divided by 4, and used as  $(R/4)$  in the formula and the time is multiplied by 4, and used as  $4T$  in the formula, given by  $A = P [ 1 + ( R / 400 ) ]^{4T}$  payable quarterly (rate =  $R/4$ , time =  $4T$ )

**Example 4:** Find the compound interest on Rs 5000 for 3 years at 6 % per annum compounded quarterly.

$$\text{Using the formula, } A = P [ 1 + ( R / 400 ) ]^{4T}$$

$$= 5000(1 + 6/400)^{3 \times 4}$$

$$= 5000 (1.015)^{12} = 5978$$

$$\text{Compound interest} = 5978 - 5000 = \text{Rs } 978$$

**3.** In case the rates are different ( $R_1, R_2, R_3 \dots$ ) for different years, the amount is given by  $P \{ 1 + R_1/100 \} \{ 1 + R_2/100 \} \{ 1 + R_3/100 \}$

**Example 5:** Find the compound interest on Rs 5000 for 3 years at 6 % per annum for first year, 7% for the second year and 8% for the third year

$$\text{Using the formula, } P \{ 1 + R_1/100 \} \{ 1 + R_2/100 \} \{ 1 + R_3/100 \}$$

$$= 5000(1 + 6/100) (1 + 7/100) (1 + 8/100)$$

$$= 6125$$

$$\text{Compound interest} = 6125 - 5000 = \text{Rs } 1125$$

**4.** For population increase the formula to be used is  $P \{ 1 + R/100 \}^T$ , and for decrease  $P \{ 1 - R/100 \}^T$ . It can also be used for depreciation factor.

**Example 6:** The death rate of a town with population of 100000 is 5 %, considering there are no new births, what is the population of town in next three years?

$$\text{Using the formula, } P \{ 1 - R/100 \}^T$$

$$= 100000(1-5/100)^3$$

$$= 100000(0.95)^3 = 85738$$

**5.** In case the period is a fraction like 3 and  $\frac{1}{2}$  years, or  $a$  and  $\frac{b}{c}$  years, then the amount should be calculated by this formula

$$A = P \{ 1 + R/100 \}^3 \{ 1 + (1/2 \times R)/100 \}$$

$$\text{Or } A = P \{ 1 + R/100 \}^a \{ 1 + (b/c \times R)/100 \}$$

**Example 7:** The birth rate of a town with population of 100000 is 5 %, considering there are no deaths in the town, what is the population of town in next three years and four months?

Three years and four months mean  $3 \frac{1}{4}$

Using the formula,  $A = P \{ 1 + R/100 \}^a \{ 1 + (b/c \times R)/100 \}$

$$= 100000(1+5/100)^3(1+ \frac{1}{4} \times 5/100)$$

$$= 100000(1.157) (1.012)$$

$$= 117210 \text{ will be the population}$$

6. The SI and CI earned during the first period remains the same.

**Example 8:** The compound interest on a certain sum of money in 2 years is 210 and the simple interest on the same amount is 200, what are the principle and the rate of interest

Since SI and CI for first year is the same, and SI for each year is the same, so SI for the first year =  $200/2 = 100$ , CI for year I = 100, that means CI for the year II =  $210 - 100 = 110$ . Here the excess of interest over year I = 10. Since the excess of interest in CI is interest over first years interest, assuming I is the interest,  $I/100 \times 100 = 10$ , so  $I = 10$ , and the principal is obviously 1000.(calculate yourself)

**Example 9:** A sum of money placed at Compound Interest doubles in every 5 years, then in how many years it will become 16 times?

Now, it is given that the principle gets doubled in every 5 years.

So, if we start from initial amount P, then in first 5 years it will become 2P.

In the next 5 years 2P will become 4P, next 5 years 4P will become 8P and finally in next 5 years 8P will become 16P.

So, it will take  $(5+5+5+5) = 20$  years.

### **Net present value (NPV)**

Money received or paid today is not the same as money received or paid after a period. This is because the money has an opportunity cost of interest in the same period. What it simply means is that you can earn interest on money if you have it now, and if you get the money later, you lose the opportunity to make interest on that. For example, if the going interest rate in the market is 10%, and someone has

to pay me Rs. 1000, and he pays after an year, so he should pay, 1100 (100 has the interest), Here 1100 is called the future value and 1000 is called the present value.

Here the Future value (FV) = Present value (PV)  $\{1 + \text{Rate}/100\}^{\text{time}}$ , which is the basic formula for amount in the case for compound interest, this is the formula to be used for calculating present value. From here

$$PV = FV / \{1 + \text{Rate}/100\}^{\text{time}}$$

This is the same formula as of the compound interest; herein we are calculating principal from the amount, which's it!

Practical applications of the NPV

### 1. Installment schemes

Today there are all kinds of loans and financing of various products right from two wheelers to houses. When a loan is taken, customer generally pays a monthly installment, his dues reduced by that amount and the interest is charged only on the balance amount which is known as reducing balance. Also there are many other concepts like floating rates etc, but they are out of purview of CAT. Here is the monthly installment formula for a fixed rate of interest (fixed means which does not change over time, floating means which changes with market conditions):

$$\text{Monthly Installment, } M = [A/(1-B)] \times P$$

Here  $A = R/1200$  (where R is the rate of interest)

$B = [1/(1+A)]^T$  (Where T is time in months)

And P is the principal amount that is the amount of loan taken

The installment can be calculated with this formula by using concept of NPV also. This formula is derived from there only. You can find this formula in Microsoft excel also under PMT in the formula section. But these annuity formula questions will not be asked in CAT.

**Example 10:** If Ram takes a home loan of 500000 for 3 years at the rate of 7.5%, what will be his monthly installment?

$$T = 12 \times 3 = 36 \text{ months}$$

$$R = 7.5\%$$

$$P = 500000$$

Using the formula,  $M = [A/(1-B)] \times P$

$$A = 7.5/1200 = 0.00625$$

$$B = [1/(1+A)]^T = [1/(1+0.00625)]^{36}$$

$$M = [0.00625 / \{1 - [1/(1+0.00625)]^{36}\}] \times 500000$$

$$M = 15553$$